Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Permutation & Combination

* भागते रहो अपने लक्ष्य के पीछे, क्यूंकि आज नहीं तो और कभी, करेंगे लोग गौर कभी, लगे रहो बस रुकना मत, आयेगा तुम्हारा दौर कभी।

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. 4 points out of 8 points in a plane are collinear. Number of different quadrilateral that can be formed by joining them is

(a) 56 (b) 53
(c) 76 (d) 60

2. Number of words of 4 letters that can be formed with the letters of the word IITJEE is

(a) 42 (b) 82
(c) 102 (d) 142

3. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals

(a) 5 (b) 7
(c) 6 (d) 4

4. The number of arrangements of the letters of the word BANANA in which the two N' s do not appear adjacently is

(a) 40 (b) 60
(c) 80 (d) 100

5. The total number of integral solution for x, y, z such that $xyz = 24$, is

(a) 3 (b) 60
(c) 90 (d) 120

6. The number of permutations of the letters a, b, c, d such that b does not follow a, and c does not follow b and d does not follow c, is

(a) 9 (b) 11
(c) 13 (d) 14

7. A class has n students, we have to form a team of the students including at least two students and also excluding at least two students. The number of ways of forming the team is

(a) $2^n - 2n$ (b) $2^n - 2n - 2$
(c) $2^n - 2n - 4$ (d) $2^n - 2n - 6$

8. Two numbers are chosen from 1, 3, 5, 7, ..., 147, 149 and 151 and multiplied together in all possible ways. The number of ways which will give us the product a multiple of 5, is

(a) 75 (b) 1020
(c) 95 (d) 1030

9. If the number of ways of selecting n cards out of unlimited number of cards bearing the number 0, 9, 3, so that they cannot be used to write the number 903 is 93, then n is equal to

(a) 3 (b) 4
(c) 5 (d) 6

10. If a, b, c are odd positive integers, then number of integral solutions of $a + b + c = 13$, is

(a) 14 (b) 21
(c) 28 (d) 56

11. A rectangle with sides $2m - 1$ and $2n - 1$ divided into squares of unit length. The number of rectangle which can be formed with sides of odd length is

(a) m^2n^2 (b) $mn(m+1)(n+1)$
(c) 4^{m+n-1} (d) none of these

12. The lock of a safe consists of five discs each of which features the digits 0, 1, 2, ..., 9. The safe can be opened by dialing a special combination of the digits. The number of days sufficient enough to open the safe, if the work day lasts 13 h and 5 s are needed to dial one combination of digits is

(a) 9 (b) 10
(c) 11 (d) 12

13. The interior angles of a regular polygon measure $160°$ each. The number of diagonals of the polygon are

(a) 97 (b) 105
(c) 135 (d) 146

14. If $a, b, c \in N$, the number of points having position vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ such that $6 \le a + b + c \le 10$ is

(a) 110 (b) 116
(c) 120 (d) 127

15. The number of 5 digit number of the form *abcba* in which $a < b$ is
(a) 320 (b) 340
(c) 360 (d) 380

16. Sum of all the odd divisors of 720 is
(a) 76 (b) 78
(c) 80 (d) 84

17. Let $A = \{x_1, x_2, x_3, x_4, x_5, x_6\}, B = \{y_1, y_2, y_3, y_4, y_5, y_6\}$. Then the number of one-one mappings from A to B such that $f(x_i) \neq y_i, i = 1, 2, 3, 4, 5, 6$ is
(a) 720 (b) 265
(c) 360 (d) 145

18. The remainder obtained, when $1! + 2! + 3! + \ldots + 175!$ is divided by 15 is
(a) 5 (b) 0
(c) 3 (d) 8

19. If the total number of m element subsets of the set $A = \{a_1, a_2, a_3, \ldots, a_n\}$ is λ times the number of 3 elements subsets containing a_4, then n is
(a) $(m-1)\lambda$ (b) $m\lambda$
(c) $(m+1)\lambda$ (d) 0

20. When simplified, the expression ${}^{47}C_4 + \sum_{n=1}^{5} {}^{52-n}C_3$ equals
(a) ${}^{47}C_5$ (b) ${}^{49}C_4$
(c) ${}^{52}C_5$ (d) ${}^{52}C_4$

21. If ${}^nC_{r-1} = 10$, ${}^nC_r = 45$ and ${}^nC_{r+1} = 120$, then r equals
(a) 1 (b) 2
(c) 3 (d) 4

22. The least positive integral value of x which satisfies the inequality ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$ is :
(a) 7 (b) 8
(c) 9 (d) 10

23. The number of diagonals that can be drawn in an octagon is
(a) 16 (b) 20
(c) 28 (d) 40

24. The number of triangles that can be formed joining the angular points of decagon, is
(a) 30 (b) 45
(c) 90 (d) 120

25. If n is an integer between 0 and 21, then the minimum value of $n!(21-n)!$ is
(a) 9 ! 2 ! (b) 10 ! 11 !
(c) 20 ! (d) 21 !

26. The maximum number of points of intersection of 8 circles, is
(a) 16 (b) 24
(c) 28 (d) 56

27. The maximum number of points of intersection of 8 straight lines, is
(a) 8 (b) 16
(c) 28 (d) 56

28. The maximum number of points into which 4 circles and 4 straight lines intersect, is
(a) 26 (b) 50
(c) 56 (d) 72

29. If 7 points out of 12 lie on the same straight line, then the number of triangles, thus formed, is
(a) 19 (b) 185
(c) 201 (d) 205

30. The total number of ways in which 9 different toys can be distributed among three different children, so that the youngest gets 4, the middle gets 3 and the oldest gets 2, is
(a) 137 (b) 236
(c) 1240 (d) 1260

31. Every one of the 10 available lamps can be switched on to illuminate certain Hall. The total number of ways in which the hall can be illuminated, is
(a) 55 (b) 1023
(c) 2^{10} (d) 10 !

32. The number of ways in which 7 persons can be seated at a round table, if two particular persons are not to sit together, is
(a) 120 (b) 480
(c) 600 (d) 720

33. The number of ways in which r letters can be posted in n letter boxes in a town, is
(a) n^r (b) r^n
(c) ${}^nP_r$ (d) ${}^nC_r$

34. The number of ways in which three students of a class may be assigned a grade of *A*, *B*, *C* or *D*, so that no two students receive the same grade, is
(a) 3^4 (b) 4^3
(c) ${}^4P_3$ (d) ${}^4C_3$

35. Six identical coins are arranged in a row. The total number of ways in which the number of heads is equal to the number of tails, is
(a) 9 (b) 20
(c) 40 (d) 120

36. If 5 parallel straight lines are intersected by 4 parallel straight lines, then the number of parallelograms, thus formed, is
(a) 20 (b) 60
(c) 101 (d) 126

37. The total number of numbers that can be formed by using all the digits 1, 2, 3, 4, 3, 2, 1, so that the odd digits always occupy the odd places, is
(a) 3 (b) 6
(c) 9 (d) 18

38. The sides *AB*, *BC* and *CA* of a triangle *ABC* have 3, 4 and 5 interior points respectively on them. The number of triangles that can be constructed using these interior points as vertices, is
(a) 205 (b) 208
(c) 220 (d) 380

39. Total number of words formed by using 2 vowels and 3 consonents taken from 4 vowels and 5 consonents is equal to

(a) 60 (b) 120
(c) 720 (d) none of these

40. Ten different letters of an alphabet are given. Words with five letters (not necessarily meaningful or pronounceable) are formed from these letters. The total number of words which have atleast one letter repeated, is

(a) 21672 (b) 30240
(c) 69760 (d) 99748

41. Twenty eight matches were played in a football tournament. Each team met its opponent only once. The number of teams that took part in the tournament, is

(a) 7 (b) 8
(c) 14 (d) none of these

42. Everybody in a room shakes hand with everybody else. The total number of handshakes is 66. The total number of persons in the room is

(a) 11 (b) 12
(c) 13 (d) 14

43. The total number of 3 digit even numbers that can be composed from the digits 1, 2, 3, ... , 9 when the repetition of digits is not allowed, is

(a) 224 (b) 280
(c) 324 (d) 405

44. The total number of 5 digit telephone numbers that can be composed with distinct digits, is

(a) ${}^{10}P_2$ (b) ${}^{10}P_5$
(c) ${}^{10}C_5$ (d) none of these

45. A car will hold 2 persons in the front seat and 1 in the rear seat. If among 6 persons only 2 can drive, the number of ways, in which the car can be filled, is

(a) 10 (b) 18
(c) 20 (d) 40

46. In an examination there are three multiple choice questions and each question has 4 choices of answers in which only one is correct. The total number of ways in which an examinee can fail to get all answers correct is

(a) 11 (b) 12
(c) 27 (d) 63

47. The sum of the digits in the unit's place of all the numbers formed with the digits 5, 6, 7, 8 when taken all at a time, is

(a) 104 (b) 126
(c) 127 (d) 156

48. Two straight lines intersect at a point O. Points $A_1, A_2, \dots, A_n$ are taken on one line and points $B_1, B_2, \dots, B_n$ on the other. If the point O is not to be used, the number of triangles that can be drawn using these points as vertices, is

(a) $n(n-1)$ (b) $n(n-1)^2$
(c) $n^2(n-1)$ (d) $n^2(n-1)^2$

49. How many different nine digit numbers can be formed from the number 22 33 55 888 by rearranging its digits, so that the odd digits occupy even positions ?

(a) 16 (b) 36
(c) 60 (d) 180

50. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$ is equal to

(a) $\binom{n+1}{r-1}$ (b) $2\binom{n+1}{r+1}$
(c) $2\binom{n+r}{r}$ (d) $\binom{n+2}{r}$

51. The number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is

(a) nine (b) eight
(c) five (d) none of these

52. A class has 21 students. The class teacher has been asked to make n groups of r students each and go to zoo taking one group at a time. The size of group (*ie*, the value of r) for which the teacher goes to the maximum number of times is (no group can go to the zoo twice)

(a) 9 or 10 (b) 10 or 11
(c) 11 or 12 (d) 12 or 13

53. The number of ways in which a score of 11 can be made from a through by three persons, each throwing a single die once, is

(a) 45 (b) 18
(c) 27 (d) 68

54. The number of positive integers with the property that they can be expressed as the sum of the cubes of 2 positive integers in two different ways is

(a) 1 (b) 100
(c) infinite (d) 0

55. The number of triangles whose vertices are the vertices of an octagon but none of whose sides happen to come from the octagon is

(a) 16 (b) 28
(c) 56 (d) 70

56. There are n different books and p copies of each in a library. The number of ways in which one or more than one book can be selected is

(a) $p^n + 1$ (b) $(p+1)^n - 1$
(c) $(p+1)^n - p$ (d) p^n

57. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no lines passes through both points A and B, and no two are parallel, then the number of intersection points the lines have is equal to

(a) 535 (b) 601
(c) 728 (d) 963

58. We are required to form different words with the help of the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begnning with I and end with R, then m_1/m_2 is given by

(a) 42 (b) 30
(c) 6 (d) 1/30

59. If a denotes the number of permutations of $x+2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x-11$ things taken all at a time such that $a = 182\,bc$, then the value of x is
(a) 15 (b) 12
(c) 10 (d) 18

60. There are n points in a plane of which no three are in a straight line except 'm' which are all in a straight line. Then the number of different quadrilaterals, that can be formed with the given points as vertices, is
(a) ${}^nC_4 - {}^mC_3\,{}^{n-m+1}C_1 - {}^mC_4$
(b) ${}^nC_4 - {}^mC_3\,{}^{n-m}C_1 + {}^mC_4$
(c) ${}^nC_4 - {}^mC_3\,{}^{n-m}C_1 - {}^mC_4$
(d) ${}^nC_4 + {}^nC_3 \cdot {}^mC_1$

61. The number of ordered triples of positive integers which are solutions of the equation $x+y+z=100$ is
(a) 5081 (b) 6005
(c) 4851 (d) 4987

62. The number of numbers less than 1000 that can be formed out of the digits, 0, 1, 2, 4 and 5, no digit being repeated is
(a) 69 (b) 68
(c) 130 (d) none of these

63. A is a set containing n elements. A subset P_1 is chosen and A is reconstructed by replacing the elements of P_1. The same process is repeated for subsets $P_1, P_2, \ldots, P_m$ with $m > 1$. The number of ways of choosing $P_1, P_2, \ldots, P_m$, so that $P_1 \cup P_2 \cup \ldots \cup P_m = A$ is
(a) $(2^m - 1)^{mn}$ (b) $(2^n - 1)^m$
(c) ${}^{m+n}C_m$ (d) none of these

64. On a railway there are 20 stations. The number of different tickets required in order that it may be possible to travel from every station to every station is
(a) 210 (b) 225
(c) 196 (d) 105

65. A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the element of P. A subset Q of A is again chosen. The number of ways of choosing P and Q, so that $P \cap Q = \phi$ is
(a) $2^{2n} - {}^{2n}C_n$ (b) 2^n
(c) $2^n - 1$ (d) 3^n

66. A father with 8 children takes 3 at a time to the zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is
(a) 336 (b) 112
(c) 56 (d) none of these

67. If the $(n+1)$ numbers $a, b, c, d, \ldots$, be all different and each of them a prime number, then the number of different factors (other than 1) of $a^m \cdot b \cdot c \cdot d \ldots$ is
(a) $m - 2^n$ (b) $(m+1)\,2^n$
(c) $(m+1)\,2^n - 1$ (d) none of these

68. The number of selections of four letters from the letters of the word ASSASSINATION is
(a) 72 (b) 71
(c) 66 (d) 52

69. The number of divisors a number 38808 can have, excluding 1 and the number itself is
(a) 70 (b) 72
(c) 71 (d) none of these

70. The letters of the word SURITI are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word SURITI is
(a) 236 (b) 245
(c) 307 (d) 315

71. The total number of seven digit numbers the sum of whose digits is even, is
(a) 9×10^6 (b) 45×10^5
(c) 81×10^5 (d) 9×10^5

72. In a steamer there are stalls for 12 animals and there are cows, horses and calves (not less than 12 of each) ready to be shipped; the total number of ways in which the ship load can be made is
(a) 3^{12} (b) 12^3
(c) ${}^{12}P_3$ (d) ${}^{12}C_3$

73. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$ is
(a) 530 (b) 532
(c) 534 (d) 536

74. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7, so that digits do not repeat and the terminal digits are even is
(a) 144 (b) 72
(c) 288 (d) 720

75. Given that n is the odd, the number of ways in which three numbers in AP can be selected from $1, 2, 3, 4, \ldots, n$ is
(a) $\dfrac{(n-1)^2}{2}$ (b) $\dfrac{(n+1)^2}{4}$
(c) $\dfrac{(n+1)^2}{2}$ (d) $\dfrac{(n-1)^2}{4}$

76. A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of chosing P and Q, so that $P \cap Q$ contains exactly two elements is
(a) $9\,{}^nC_2$ (b) $3^n - {}^nC_2$
(c) $2\,{}^nC_n$ (d) none of these

77. Eight straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. The number of parts into which these lines divide the plane, is
(a) 29 (b) 32
(c) 36 (d) 37

78. Number of divisors of the form $4n+2\,(n \geq 0)$ of the integer 240 is
(a) 4 (b) 8
(c) 10 (d) 3

79. An n digit number is a positive number with exactly n digits. Nine hundred distinct n digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is
(a) 6 (b) 7
(c) 8 (d) 9

80. If a, b, c, d are odd natural numbers such that $a + b + c + d = 20$, then the number of values of the ordered quadruplet (a, b, c, d) is
(a) 165 (b) 310
(c) 295 (d) 398

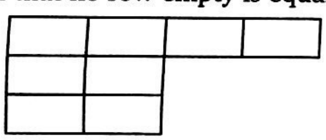
81. The number of rectangles excluding squares from a rectangle of size 15×10 is
(a) 3940 (b) 4940
(c) 5940 (d) 6940

82. In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, 3, 4, \ldots, n$. If the total number of wrong answers given is 2047, then n is equal to
(a) 10 (b) 11
(c) 12 (d) 13

83. The exponent of 3 in 100 ! is
(a) 12 (b) 24
(c) 48 (d) 96

84. If all permutations of the letters of the word AGAIN are arranged as in dictionary, then fiftieth word is
(a) NAAGI (b) NAGAI
(c) NAAIG (d) NAIAG

85. The number of different ways the letters of the word VECTOR can be placed in the 8 boxes of the given below such that no row empty is equal to



(a) 26 (b) $26 \times 6!$
(c) $6!$ (d) $2! \times 6!$

86. In the next world cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will qualify for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, when each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next world cup will be
(a) 54 (b) 53
(c) 52 (d) none of these

87. Two lines intersect at O. Points $A_1, A_2, \ldots, A_n$ are taken on one of them and $B_1, B_2, \ldots, B_n$ on the other the number of triangles that can be drawn with the help of these $(2n + 1)$ points is
(a) n (b) n^2
(c) n^3 (d) n^4

88. Seven different lecturers are to deliver lectures in seven periods of a class on a particular day. A, B and C are three of the lectures. The number of ways in which a routine for the day can be made such that A delivers his lecture before B, and B before C, is
(a) 210 (b) 420
(c) 840 (d) none of these

89. If 33 ! is divisible by 2^n, then the maximum value of n is equal to
(a) 33 (b) 32
(c) 31 (d) 30

90. The number of zeros at the end of 100 ! is
(a) 54 (b) 58
(c) 24 (d) 47

91. The maximum number of different permutations of 4 letters of the word EARTHQUAKE is
(a) 1045 (b) 2190
(c) 4380 (d) 2348

92. In a city no persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is
(a) 2^{32} (b) $2^{32} - 1$
(c) $2^{32} - 2$ (d) $2^{32} - 3$

93. The number of ways in which a mixed double game can be arranged from amongst 9 married couples, if no husband and wife play in the same game is
(a) 756 (b) 1512
(c) 3024 (d) none of these

94. In a college examination, a candidate is required to answer 6 out of 10 questions which are divided into two sections each containing 5 questions further the candidate is not permitted to attempt more than 4 questions from either of the section. The number of ways in which he can make up a choice of 6 questions is
(a) 200 (b) 150
(c) 100 (d) 50

95. The number of ways in which 9 identical balls can be placed in three identical boxes is
(a) 55 (b) $\frac{9!}{(3!)^4}$
(c) $\frac{9!}{(3!)^3}$ (d) 12

96. If the number of arrangements of $(n - 1)$ things taken from n different things is k times the number of arrangements of $(n - 1)$ things taken from n things in which two things are identical, then the value of k is
(a) 1/2 (b) 2
(c) 4 (d) none of these

97. The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(a) ${}^7P_2 \cdot 2^5$ (b) ${}^7C_2 \cdot 2^5$
(c) ${}^7C_2 \cdot 5^2$ (d) none of these

98. The number of points (x, y, z) in space, whose each coordinate is a negative integer such that $x + y + z + 12 = 0$ is

(a) 385 (b) 55
(c) 110 (d) none of these

99. The number of divisors of $2^2 \cdot 3^3 \cdot 5^3 \cdot 7^5$ of the form $4n + 1, n \in N$ is

(a) 46 (b) 47
(c) 96 (d) 94

100. The number of ways in which 30 coins of one rupee each be given to six persons, so that none of them receives less than 4 rupees is

(a) 231 (b) 462
(c) 693 (d) 924

101. The number of integral solutions of the equation $2x + 2y + z = 20$, where $x \geq 0$, $y \geq 0$ and $z \geq 0$ is

(a) 132 (b) 11
(c) 33 (d) 66

102. The number of ways in which we can choose 2 distinct integers from 1 to 100 such that difference between them is at most 10 is

(a) ${}^{10}C_2$ (b) 72
(c) ${}^{100}C_2 - {}^{90}C_2$ (d) none of these

103. Number of points having position vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ where $a, b, c \in \{1, 2, 3, 4, 5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is

(a) 70 (b) 140
(c) 210 (d) 280

104. Number of positive integral solutions of $xyz = 30$ is

(a) 9 (b) 27
(c) 81 (d) 243

Answers

Objective Questions Type I [Only one correct answer]

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(d)	**6.**	(b)	**7.**	(b)	**8.**	(b)	**9.**	(c)	**10.**	(b)
11.	(a)	**12.**	(c)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(b)	**18.**	(c)	**19.**	(b)	**20.**	(d)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(d)	**25.**	(b)	**26.**	(d)	**27.**	(c)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(b)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(b)	**36.**	(b)	**37.**	(d)	**38.**	(a)	**39.**	(d)	**40.**	(c)
41.	(b)	**42.**	(b)	**43.**	(a)	**44.**	(b)	**45.**	(d)	**46.**	(d)	**47.**	(d)	**48.**	(c)	**49.**	(c)	**50.**	(d)
51.	(b)	**52.**	(b)	**53.**	(c)	**54.**	(c)	**55.**	(a)	**56.**	(b)	**57.**	(a)	**58.**	(b)	**59.**	(b)	**60.**	(c)
61.	(c)	**62.**	(b)	**63.**	(d)	**64.**	(a)	**65.**	(d)	**66.**	(c)	**67.**	(c)	**68.**	(a)	**69.**	(a)	**70.**	(a)
71.	(b)	**72.**	(a)	**73.**	(d)	**74.**	(d)	**75.**	(d)	**76.**	(d)	**77.**	(d)	**78.**	(a)	**79.**	(b)	**80.**	(a)
81.	(c)	**82.**	(b)	**83.**	(c)	**84.**	(c)	**85.**	(b)	**86.**	(b)	**87.**	(c)	**88.**	(c)	**89.**	(c)	**90.**	(c)
91.	(b)	**92.**	(b)	**93.**	(b)	**94.**	(a)	**95.**	(d)	**96.**	(b)	**97.**	(b)	**98.**	(b)	**99.**	(b)	**100.**	(b)
101.	(d)	**102.**	(c)	**103.**	(a)	**104.**	(b)												